What Happens If an Unbroken Flavor Symmetry Exists?

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Abstract

Without assuming any specific flavor symmetry and/or any specific mass matrix forms, it is demonstrated that if a flavor symmetry (a discrete symmetry, a U(1) symmetry, and so on) exists, we cannot obtain the CKM quark mixing matrix V and the MNS lepton mixing matrix U except for those between two families for the case with the completely undegenerated fermion masses, so that we can never give the observed CKM and MNS mixings. Only in the limit of $m_{\nu 1} = m_{\nu 2}$ ($m_d = m_s$), we can obtain three family mixing with an interesting constraint $U_{e3} = 0$ ($V_{ub} = 0$).

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1. Introduction

It is well known that the masses of the charged fermions rapidly increase as $(u, d, e) \to (c, s, \mu) \to (t, b, \tau)$. It has been considered that the rapid increasing of the mass spectra cannot be understood from an idea of "symmetry". The horizontal degree of freedom has been called as "generations". In contrast to the idea of "generations", there is an idea of "families" that the horizontal quantum number states have basically the same opportunity. It is after the democratic mass matrix model [1] was proposed that the idea of "families" became one of the promising viewpoints for "flavors". Nowadays, a popular idea to understand the observed quark and lepton mass spectra and mixing matrices is to assume a flavor symmetry which puts constraints on the Yukawa coupling constants.

In the present paper, we will point out that if a flavor symmetry (a discrete symmetry, a U(1) symmetry, and so on) exists, we cannot obtain the observed Cabibbo-Kobayashi-Maskawa [2] (CKM) quark mixing matrix V_q and Maki-Nakagawa-Sakata [3] (MNS) lepton mixing matrix U_ℓ, even if we can obtain reasonable mass spectra under the symmetry. You may think that this conclusion is not so remarkable and rather trivial, because anyone thinks that the flavor symmetry is badly broken. However, most investigations on the broken flavor symmetries are based on specific models, and we are not clearly aware that what problem happens if a flavor symmetry, in general, exists until a low energy scale $\mu \sim 10^2$ GeV. In the present paper, without assuming any explicit flavor symmetry and/or any explicit mass matrix forms, we will demonstrate how it is serious.

Even when we consider a broken flavor symmetry, it is important to consider the world in which the flavor symmetry is unbroken. In the present paper, we will conclude that in such a world with an unbroken flavor symmetry, the CKM and MNS mixing matrices cannot describe flavor mixings except for those between two families when the fermion masses are completely different from each other, and that only when $m_{\nu 1} = m_{\nu 2}$ ($m_d = m_s$), the MNS matrix U_ℓ (the CKM matrix V_q) can describe a three family mixing with an interesting constraint $(U_\ell)_{e3} = 0$ ($(V_q)_{ub} = 0$). This will suggests that our world with a broken flavor symmetry should be derived from what unbroken world.

In the derivation of the conclusion, a requirement that the SU(2)$_L$ symmetry must not be broken plays an essential role. Generally, the terminology "symmetry" can have a meaning only by defining the world

in which the symmetry is exactly unbroken. In some of phenomenological mass matrix models, flavor symmetry breaking terms are brought into the theory by hand, and it is not clear whether the terms can be generated without breaking the $\mathrm{SU}(2)_L$ or not. In the present paper, we regard such a model with an ad hoc flavor symmetry breaking as a model without a flavor symmetry, and we will comment only on a model where the $\mathrm{SU}(2)_L$ symmetry is exactly unbroken at the original Lagrangian, and the flavor symmetry breaking mechanism does not spoil the $\mathrm{SU}(2)_L$ invariance.

First, let us consider that the up- and down-quark fields transform under a flavor symmetry as

$$\begin{aligned} u_L &= U^u_{XL} u'_L, \quad u_R = U^u_{XR} u'_R, \\ d_L &= U^d_{XL} d'_L, \quad d_R = U^d_{XR} d'_R. \end{aligned} \tag{1}$$

If the Lagrangian is invariant under the transformation (1), the Yukawa coupling constants Y_u and Y_d must satisfy the relations

$$(U^u_{XL})^\dagger Y_u U^u_{XR} = Y_u, \quad (U^d_{XL})^\dagger Y_d U^d_{XR} = Y_d, \tag{2}$$

where $U^u_{XL}(U^u_{XL})^\dagger = \mathbf{1}$, and so on. Since these transformations must not break $\mathrm{SU}(2)_L$ symmetry, we cannot consider a case with $U^u_{XL} \neq U^d_{XL}$. We must rigorously take

$$U^u_{XL} = U^d_{XL} \equiv U_X. \tag{3}$$

Therefore, the up- and down-quark mass matrices $M_u = Y_u \langle H^0_u \rangle$ and $M_d = Y_d \langle H^0_d \rangle$ must satisfy the relations

$$U^\dagger_X M_u M^\dagger_u U_X = M_u M^\dagger_u, \quad U^\dagger_X M_d M^\dagger_d U_X = M_d M^\dagger_d, \tag{4}$$

independently of U^u_{XR} and U^d_{XR}.

Similar situation is required in the lepton sectors. Although, sometimes, in the basis where the charged lepton mass matrix M_e is diagonal (i.e. $M_e = D_e \equiv \mathrm{diag}(m_e, m_\mu, m_\tau)$), a "symmetry" for the neutrino mass matrix M_ν is investigated, such a prescription cannot be regarded as a field theoretical symmetry. For example, when we assume a permutation symmetry between neutrinos ν_{L2} and ν_{L3}, we can obtain a nearly bimaximal mixing [4]. However, the symmetry is applied only to neutrino sector M_ν, and not to the charged lepton sector $M_e = D_e$. Therefore, we cannot regard this $2 \leftrightarrow 3$ permutation rule as a "symmetry" in the field theoretical meaning, because it is badly broken the $\mathrm{SU}(2)_L$ symmetry.

In the lepton sectors, we must consider that under the transformations

$$\begin{aligned} \nu_L &= U_X \nu'_L, \quad \nu_R = U^\nu_{XR} \nu'_R, \\ e_L &= U_X e'_L, \quad e_R = U^e_{XR} e'_R, \end{aligned} \tag{5}$$

the Yukawa coupling constants which are defined by $\overline{e}_L Y_e e_R$, $\overline{\nu}_L Y^\nu_D e_R$, and $\overline{\nu}^c_R Y^\nu_M \nu_R$ ($\nu^c_R \equiv C\overline{\nu}^T_R$) are invariant as follows

$$\begin{aligned} U^\dagger_X Y_e U^e_{XR} &= Y_e, \\ U^\dagger_X Y^\nu_D U^\nu_{XR} &= Y^\nu_D, \\ U^T_{XR} Y^\nu_M U_{XR} &= Y^\nu_M. \end{aligned} \tag{6}$$

In other words, the mass matrices $M_e M_e^\dagger$ and M_ν are invariant under the transformation U_X as

$$U^\dagger_X M_e M^\dagger_e U_X = M_e M^\dagger_e, \tag{7}$$

$$U^\dagger_X M_\nu U^*_X = M_\nu, \tag{8}$$

independently of the forms U^ν_{XR} and U^e_{XR}, where we assumed the seesaw mechanism [5] $M_\nu \propto Y^\nu_D (Y^\nu_M)^{-1} (Y^\nu_D)^T$. (Even when we do not assume the seesaw mechanism, as long as the effective neutrino mass matrix is given by $\overline{\nu}_L M_\nu \nu^c_L$, the mass matrix must obey the constraint (8).)

Note that the constraints (4) [and also (7) and (8)] do not always mean that the matrix forms $M_u M_u^\dagger$ and $M_d M_d^\dagger$ are identical each other. Indeed, in the present paper, we consider a general case in which the eigenvalues and mixing matrices between $M_u M_u^\dagger$ and $M_d M_d^\dagger$ are different from each other. Nevertheless, the conditions (4) [and also (7) and (8)] will put very strong constraints on the CKM mixing matrix $V_q = (U^u_L)^\dagger U^d_L$ [and also the MNS mixing matrix $U_\ell = (U^e_L)^\dagger U^\nu_L$], where U^f_L $(f = u, d, e, \nu)$ are defined by

$$(U^f_L)^\dagger M_f M_f^\dagger U^f_L = D_f^2 \equiv \mathrm{diag}(m^2_{f1}, m^2_{f2}, m^2_{f3}) \quad (f = u, d, e), \tag{9}$$

$$(U^\nu_L)^\dagger M_\nu (U^\nu_L)^* = D_\nu \equiv \mathrm{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}). \tag{10}$$

The purpose of the present paper is to see whether it is possible or not to consider such the flavor symmetry without an $\mathrm{SU}(2)_L$ symmetry breaking. Of course, further conditions

$$(U^f_{XR})^\dagger M_f^\dagger M_f U^f_{XR} = M_f^\dagger M_f \quad (f = u, d, e), \tag{11}$$

will give more strict constraints on the mass matrices M_f. However, even apart form such an additional constraint, by using only the constraints (4), (7) and (8), we will obtain a severe conclusion that such a symmetry cannot lead to the observed CKM mixing matrix V_q and MNS mixing matrix U_ℓ.

2. Trouble in the CKM and MNS mixing matrices

First, we investigate relations in the quark sectors under the conditions (4). Since we can rewrite the left hand of Eq. (9) by using Eq. (4) as

$$(U^f_L)^\dagger M_f M_f^\dagger U^f_L = (U^f_L)^\dagger U_X^\dagger M_f M_f^\dagger U_X U^f_L = (U^f_L)^\dagger U_X^\dagger U^f_L D_f^2 (U^f_L)^\dagger U_X U^f_L, \tag{12}$$

for $f = u, d$, we obtain the relation

$$(U^f_X)^\dagger D_f^2 U^f_X = D_f^2, \tag{13}$$

where

$$U^f_X = (U^f_L)^\dagger U_X U^f_L. \tag{14}$$

Therefore, the matrix U^f_X which satisfies Eq. (13) must be a diagonal matrix with a form

$$U^f_X = P^f_X \equiv \mathrm{diag}(e^{i\delta^f_1}, e^{i\delta^f_2}, e^{i\delta^f_3}), \tag{15}$$

unless the masses are not degenerated. Therefore, from (14), we obtain

$$U_X = U^u_L P^u_X (U^u_L)^\dagger = U^d_L P^d_X (U^d_L)^\dagger, \tag{16}$$

which leads to a constraint on the CKM matrix $V_q \equiv (U^u_L)^\dagger U^d_L$:

$$P^u_X = V_q P^d_X (V_q)^\dagger. \tag{17}$$

The constraint (17) (i.e. $P^u_X V_q = V_q P^d_X$) requires

$$(e^{i\delta^u_i} - e^{i\delta^d_j})(V_q)_{ij} = 0 \quad (i, j = 1, 2, 3). \tag{18}$$

Only when $\delta_i^u = \delta_j^d$, we can obtain $(V_q)_{ij} \neq 0$. For the case $\delta_1^u = \delta_2^u = \delta_3^u \equiv \delta_u$ (also $\delta_1^d = \delta_2^d = \delta_3^d \equiv \delta_d$), the matrix $P_X^u = \mathbf{1}e^{i\delta_u}$ (and also $P_X^d = \mathbf{1}e^{i\delta_d}$) leads to a trivial result $U_X = \mathbf{1}e^{i\delta_u} = \mathbf{1}e^{i\delta_d}$), so that we do not consider such a case. Therefore, from the requirement (17), we cannot consider such a case as all elements of V_q are not zero. For example, if we can take $(V_q)_{ii} \neq 0$ for $i = 1, 2, 3$ by taking $\delta_i^u = \delta_i^d \equiv \delta_i$, since we must choose, at least, one of δ_i differently from others, we obtain a mixing matrix between only two families, e.g. $(V_q)_{13} = (V_q)_{31} = (V_q)_{23} = (V_q)_{32} = 0$ for a case of $\delta_1 = \delta_2 \neq \delta_3$:

$$V = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{19}$$

Thus, for any choice of δ_i^u and δ_j^d, the condition (18) cannot lead to the observed CKM mixing matrix.

For the lepton sectors, the situation is the same. From Eqs. (8) and (10), we obtain the constraint

$$(U_X^\nu)^\dagger D_\nu (U_X^\nu)^* = D_\nu, \tag{20}$$

where

$$U_X^\nu = (U_L^\nu)^\dagger U_X U_L^\nu. \tag{21}$$

Again, if we assume that the neutrino masses are not degenerated, we obtain that the matrix U_X^ν must be diagonal, and it is given by

$$U_X^\nu = P_X^\nu \equiv \mathrm{diag}(e^{i\delta_1^\nu}, e^{i\delta_2^\nu}, e^{i\delta_3^\nu}), \tag{22}$$

because the constraint (20) leads to

$$\left(m_{\nu i} e^{-i\phi_{ij}} - m_{\nu j} e^{i\phi_{ij}}\right) |(U_X^\nu)_{ij}| = 0, \tag{23}$$

where we have put $(U_X^\nu)_{ij} = |(U_X^\nu)_{ij}| e^{i\phi_{ij}}$. Here, differently from the matrix (15), the phases δ_i^ν are constrained as $\delta_i^\nu = 0$ or $\delta_i^\nu = \pi$ $(i = 1, 2, 3)$ from the condition (20). From the relations (14) and (21), we obtain

$$U_X = U_L^e P_X^e (U_L^e)^\dagger = U_L^\nu P_X^\nu (U_L^\nu)^\dagger, \tag{24}$$

so that the MNS matrix $U_\ell = (U_L^e)^\dagger U_L^\nu$ must satisfy the constraint

$$P_X^e = U_\ell P_X^\nu (U_\ell)^\dagger, \tag{25}$$

i.e.

$$(e^{i\delta_i^e} - e^{i\delta_j^\nu})(U_\ell)_{ij} = 0 \quad (i, j = 1, 2, 3). \tag{26}$$

Again, only when $\delta_i^e = \delta_j^\nu$, we can obtain $(U_\ell)_{ij} \neq 0$, and we cannot consider a case in which all elements of U_ℓ are not zero. We only obtain a mixing matrix between two families.

Thus, the requirements (4) [and also (7) and (8)] lead to a serious trouble in the CKM matrix V_q (the MNS matrix U_ℓ), even if we can suitably give the observed mass spectra. The similar conclusion has already been derived by Low and Volkas [6] although they have demonstrated it by using explicit mass matrix forms.

3. Should we abandon any flavor symmetry?

In order to evade the conclusion (18) [and also the conclusion (26)], we may consider a case with $U^u_{XL} \neq U^d_{XL}$ $[U^e_{XL} \neq U^\nu_{XL}]$. However, such a transformation breaks SU$(2)_L$, so that it is highly unrealistic.

If there is no symmetry breaking term in the original Lagrangian, even if we take the renormalization group equation (RGE) effects into consideration, the SU$(2)_L$ is never broken, and the relations (4), (7) and (8) are still unchanged.

If we consider a U(1) charge model, we cannot assign different charges to u_{Li} and d_{Li} [and also to ν_{Li} and e_{Li}], so that we must take the operator U_X as

$$U^u_{XL} = U^d_{XL} \equiv U_X = \mathrm{diag}(e^{iQ_1\theta}, e^{iQ_2\theta}, e^{iQ_3\theta}). \tag{27}$$

In this case, since the Higgs scalars H_u and H_d can have different charges, the mass terms $\overline{u}_L M_u u_R H_u$ and $\overline{d}_L M_d d_R H_d$ can have different phases for the transformation. However, since the additional phases form Higgs sector are common for all flavors, the conclusion (18) is essentially unchanged.

Related to an extended version of the U(1) charge model, we know the Froggatt an Nielsen model [7]. The model can evade the present constraints (18) and (26). In this model, each flavor state at a low energy scale has a different hierarchical structure, so that the fermion flavors are ones which should be understood from the concept of "generations" rather than from that of "families". The constraints in the present paper cannot be applied to a model with "generation" structures, and the Froggatt and Nielsen model is indeed one of the most promising models which can reasonably understand the generations.

Thus, it is one way to adopt a model with no flavor symmetry in order to evade the present severe conclusions (18) and (26). However, we know the fact (the degree of freedom of "rebasing") that we cannot physically distinguish two mass matrix sets (M_u, M_d) and (M'_u, M'_d), where (M'_u, M'_d) is obtained from (M_u, M_d) by a common flavor-basis rotation for the SU$(2)_L$ doublet fields. (The situation is the same in the lepton sector.) Only when there is a flavor symmetry, the mass matrix forms (M_u, M_d) in a specific flavor basis have a meaning, because the operator of the flavor rotation does not commute with the flavor symmetry operator U_X. Therefore, the idea of a flavor symmetry is still attractive to most mass-matrix-model-builders.

4. Case of $m_{\nu 1} = m_{\nu 2}$

In order to seek for a clue to a possible symmetry breaking, let us go on a phenomenological study.

Since the observed neutrino data [8, 9, 10, 11] have shown $\Delta m^2_{solar} \ll \Delta m^2_{atm}$, it is interesting to consider a limit of $m_{\nu 1} = m_{\nu 2}$. In this case, the conclusion (22) [and (26)] is not correct any more, because the constraint (20) allows a case with $(U^\nu_X)_{12} \neq 0$ and $(U^\nu_X)_{21} \neq 0$:

$$U^\nu_X = \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{28}$$

or

$$U^\nu_X = \begin{pmatrix} -c & s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{29}$$

where $c = \cos\theta$ and $s = \sin\theta$. (Again, each element must be real.) Therefore, we must check the relation

$$P_X^e = U_\ell U_X^\nu (U_\ell)^\dagger, \tag{30}$$

with the forms of (28) and (29) of U_X^ν, instead of (22).

Now, we explicitly calculate $U_\ell U_X^\nu (U_\ell)^\dagger$ by using a general form of U_ℓ

$$U_\ell = V P_M, \tag{31}$$

where

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{32}$$

and P_M is a Majorana phase matrix

$$P_M = \mathrm{diag}(e^{i\alpha}, e^{i\beta}, e^{i\gamma}). \tag{33}$$

For the case with the form (28) of U_X^ν, we obtain

$$\begin{aligned}(U_\ell U_X^\nu U_\ell^\dagger)_{12} &= -c_{13}\left\{c_{23}\left(c_{12}^2 e^{-i\phi} + s_{12}^2 e^{i\phi}\right)s \right.\\ &\quad \left. + s_{13}s_{23}\left[(e^{i\phi} - e^{-i\phi})c_{12}s_{12}s + c - 1\right]e^{-i\delta}\right\}\end{aligned} \tag{34}$$

$$\begin{aligned}(U_\ell U_X^\nu U_\ell^\dagger)_{13} &= c_{13}\left\{s_{23}\left(c_{12}^2 e^{-i\phi} + s_{12}^2 e^{i\phi}\right)s \right.\\ &\quad \left. - s_{13}c_{23}\left[(e^{i\phi} - e^{-i\phi})c_{12}s_{12}s + c - 1\right]e^{-i\delta}\right\}\end{aligned} \tag{35}$$

$$\begin{aligned}(U_\ell U_X^\nu U_\ell^\dagger)_{23} &= c_{23}s_{23}\left[(e^{i\phi} - e^{-i\phi})c_{12}s_{12}(1 + s_{13}^2)s + c_{13}^2(1 - c)\right] \\ &\quad - s_{13}s\left[s_{23}^2(c_{12}^2 e^{-i\phi} + s_{12}^2 e^{i\phi})e^{i\delta} + c_{23}^2(c_{12}^2 e^{i\phi} + s_{12}^2 e^{-i\phi})e^{-i\delta}\right],\end{aligned} \tag{36}$$

where $\phi = \beta - \alpha$. If $c_{13} \neq 0$, there is no solution which gives zeros for all the elements (34) – (36), except for a trivial solution with $c = 1$ (i.e. $U_X = \mathbf{1}$). If $c_{13} = 0$, there is a solution for suitable choice of ϕ and δ, and then, the matrix V takes the form

$$V = \begin{pmatrix} 0 & 0 & e^{-i\delta} \\ * & * & 0 \\ * & * & 0 \end{pmatrix}. \tag{37}$$

Of course, the form (37) is ruled out. Thus, the case (28) cannot lead to any interesting form of U_ℓ.

On the other hand, for the case (29), we obtain

$$\begin{aligned}(U_\ell U_X^\nu U_\ell^\dagger)_{12} &= c_{13}\left\{c_{23}\left[(c_{12}^2 e^{-i\phi} - s_{12}^2 e^{i\phi})s + 2c_{12}s_{12}c\right] \right.\\ &\quad \left. + s_{13}s_{23}\left[1 + (c_{12}^2 - s_{12}^2)c - (e^{i\phi} + e^{-i\phi})c_{12}s_{12}s\right]e^{-i\delta}\right\}\end{aligned} \tag{38}$$

$$\begin{aligned}(U_\ell U_X^\nu U_\ell^\dagger)_{13} &= -c_{13}\left\{s_{23}\left[(c_{12}^2 e^{-i\phi} - s_{12}^2 e^{i\phi})s + 2c_{12}s_{12}c\right] \right.\\ &\quad \left. - s_{13}c_{23}\left[1 + (c_{12}^2 - s_{12}^2)c - (e^{i\phi} + e^{-i\phi})c_{12}s_{12}s\right]e^{-i\delta}\right\}\end{aligned} \tag{39}$$

$$
\begin{aligned}
(U_\ell U_X^\nu U_\ell^\dagger)_{23} &= c_{23}s_{23}\left[(e^{i\phi}+e^{-i\phi})c_{12}s_{12}(1+s_{13}^2)s+c_{13}^2-(c_{12}^2-s_{12}^2)(1+s_{13}^2)c\right] \\
&\quad +s_{13}s_{23}^2\left[(c_{12}^2e^{-i\phi}-s_{12}^2e^{i\phi})s+2c_{12}s_{12}c\right]e^{i\delta} \\
&\quad -s_{13}c_{23}^2\left[(c_{12}^2e^{i\phi}-s_{12}^2e^{-i\phi})s+2c_{12}s_{12}c\right]e^{-i\delta}. \qquad (40)
\end{aligned}
$$

The case can lead to a non-trivial solution for $s_{13}=0$, $\phi=\beta-\alpha=0$ and

$$
\cos(2\theta_{12}+\theta)=1, \qquad (41)
$$

i.e.

$$
U_\ell = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -c_{23}s_{12} & c_{23}c_{12} & s_{23} \\ s_{23}s_{12} & -s_{23}c_{12} & c_{23} \end{pmatrix} P_M. \qquad (42)
$$

It should be noted that in the limit of $m_{\nu 1}=m_{\nu 2}$, the Majorana phases in P_M must be $\alpha=\beta$.

The similar result $(U_\ell)_{13}=0$ has also been derived by Low and Volkas [6] although their interest was in the "trimaximal mixing" and they have assumed a specific flavor symmetry. In the present general study, we can obtain $s_{13}=0$, but s_{12} and s_{23} are still free. The result (42) is a conclusion which is derived model-independently.

Note that the case (29) satisfies $(U_X^\nu)^2=\mathbf{1}$, so that the flavor transformation U_X also satisfies

$$
(U_X)^2=\mathbf{1}. \qquad (43)
$$

This suggests that an approximate flavor symmetry in the lepton sectors is a discrete symmetry Z_2.

Inversely, for the neutrino mass spectra with $m_{\nu 1}\neq m_{\nu 2}$, if we take the operator $U_X=U_L^\nu U_X^\nu (U_L^\nu)^\dagger$ with the form (29) of U_X^ν, we obtain

$$
(U_X^\nu)^\dagger D_\nu (U_X^\nu)^* = D_\nu + (m_{\nu 2}-m_{\nu 1})s\begin{pmatrix} s & c & 0 \\ c & -s & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (44)
$$

which leads to

$$
U_X^\dagger M_\nu U_X^* = M_\nu + (m_{\nu 2}-m_{\nu 1})sB, \qquad (45)
$$

where the symmetry breaking term B is given by

$$
B = U_L^\nu \begin{pmatrix} s & c & 0 \\ c & -s & 0 \\ 0 & 0 & 0 \end{pmatrix} (U_L^\nu)^T. \qquad (46)
$$

The matrix B is rewritten as

$$
B = U_L^e \begin{pmatrix} 0 & c_{23} & -s_{23} \\ c_{23} & 0 & 0 \\ -s_{23} & 0 & 0 \end{pmatrix} (U_L^e)^\dagger, \qquad (47)
$$

by using the relation $U_X = U_L^e P_X^e (U_X^e)^\dagger$ and the constraint (41). Of course, the result (45) shows that in the limit of $m_{\nu 1} = m_{\nu 2}$ and/or $s = 0$, the operation U_X becomes that of the exact symmetry. The forms (46) and (47) of the symmetry breaking term will give a clue to a possible form of the flavor symmetry breaking. However, in order to fix the values of s_{23} and s_{12} (or s), we must put a further assumption. In the present paper, we do not give such a speculation any more.

If we apply the similar discussion to the quark sector in the limit of $m_d = m_s$, we can obtain $|V_{ub}| = 0$. This may be taken as the reason of $|V_{ub}|^2 \ll |V_{cb}|^2, |V_{us}|^2$.

5. Concluding remarks

In conclusion, we have noticed that when we assume a flavor symmetry, we must use the same operation U_X simultaneously for the up-quarks u_{Li} and down-quarks d_{Li} (and also for the charged leptons e_{Li} and neutrinos ν_{Li}), and we have demonstrated that the existence of such an operation U_X without an $\mathrm{SU}(2)_L$ breaking leads to unwelcome forms of the CKM mixing matrix $V_q = (U_L^u)^\dagger U_L^d$ and the MNS mixing matrix $U_\ell (U_L^e)^\dagger U_L^\nu$, even if we can obtain reasonable mass spectra: in the limit of an unbroken flavor symmetry, the CKM and MNS mixing matrices cannot describe flavor mixings except for only those between two families when the fermion masses are completely different from each other, and that only when $m_{\nu 1} = m_{\nu 2}$ ($m_d = m_s$), the MNS matrix U_ℓ (the CKM matrix V_q) can describe a three family mixing with an interesting constraint $(U_\ell)_{e3} = 0$ ($(V_q)_{ub} = 0$).

If we want to investigate the "generation" problem from the standpoint of flavor symmetry, our results (18) and (26) demands that the flavor symmetry should be completely broken at a high energy scale M_X, so that we cannot have any flavor symmetry below $\mu = M_X$. We have to seek for a flavor symmetry breaking mechanism under the condition that the original Lagrangian (including the symmetry breaking mechanism) is exactly invariant under the $\mathrm{SU}(2)_L$.

For example, let us consider a two Higgs doublet model, or a $\overline{5}_L \leftrightarrow \overline{5}'_L$ model [12]. In such a model, the effective Yukawa coupling constants Y^f below $\mu = M_X$ are given by a linear combination of two Yukawa coupling constants with different textures Y_A^f and Y_B^f,

$$Y^f = c_A^f Y_A^f + c_B^f Y_B^f, \tag{48}$$

so that $Y^f (Y^f)^\dagger$ do not satisfy the flavor symmetry condition

$$U_X^\dagger Y^f (Y^f)^\dagger U_X = Y^f (Y^f)^\dagger, \tag{49}$$

although $Y_A^f (Y_A^f)^\dagger$ and $Y_B^f (Y_B^f)^\dagger$ must satisfy the conditions

$$U_X^\dagger Y_A^f (Y_A^f)^\dagger U_X = Y_A^f (Y_A^f)^\dagger, \quad U_X^\dagger Y_B^f (Y_B^f)^\dagger U_X = Y_B^f (Y_B^f)^\dagger, \tag{50}$$

respectively, even if at $\mu < M_X$. In other words, there is no operator U_X which satisfies the condition (49). Thus, we can break the flavor symmetry without the $\mathrm{SU}(2)_L$ symmetry.

However, we should note that the matrices Y_A^f and Y_B^f have to satisfy the conditions (50). As an example, let see a two Higgs doublet model with Z_3 and S_2 symmetries [13]. In the model, we assume that under a discrete symmetry Z_3, the quark and lepton fields ψ_L, which belong to 10_L, $\overline{5}_L$ and 1_L of SU(5) ($1_L = \nu_R^c$), are transformed as

$$\psi_{1L} \to \psi_{1L}, \quad \psi_{2L} \to \omega \psi_{2L}, \quad \psi_{3L} \to \omega \psi_{3L}, \tag{51}$$

where $\omega = e^{2i\pi/3}$. [Although we use a terminology of SU(5), at present, we do not consider the SU(5) grand unification.] Then, the bilinear terms $\overline{q}_{Li} u_{Rj}$, $\overline{q}_{Li} d_{Rj}$, $\overline{\ell}_{Li} \nu_{Rj}$, $\overline{\ell}_{Li} e_{Rj}$ and $\overline{\nu}_{Ri}^c \nu_{Rj}$ [$\nu_R^c = (\nu_R)^c =$

$C\overline{\nu_R}^T$ and $\overline{\nu}_R^c = \overline{(\nu_R^c)}$] are transformed as follows:

$$\begin{pmatrix} 1 & \omega^2 & \omega^2 \\ \omega^2 & \omega & \omega \\ \omega^2 & \omega & \omega \end{pmatrix} . \tag{52}$$

Therefore, if we assume two SU(2) doublet Higgs scalars H_A and H_B, which are transformed as

$$H_A \to \omega H_A, \quad H_B \to \omega^2 H_B, \tag{53}$$

we obtain Yukawa coupling constants with the following textures:

$$Y_A^f = a_f \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad Y_B^f = b_f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & x_f \\ 0 & x_f & 1 \end{pmatrix}, \tag{54}$$

where in addition to the Z_3 symmetry, we have assumed a flavor $2 \leftrightarrow 3$ symmetry (S_2 symmetry). At a high energy scale $\mu = M_X$, the Z_3 symmetry is broken, and the mixing between H_A and H_B takes place. We assume that the one component of the linear combinations of H_A and H_B plays a role of the conventional Higgs scalar, so that we obtain the following universal texture of quark and lepton mass matrices M_f:

$$M_f = P_f \widehat{M}_f P_f \ , \tag{55}$$

where P_f is a phase matrix defined by

$$P_f = \mathrm{diag}(e^{i\delta_1^f},\ e^{i\delta_2^f},\ e^{i\delta_3^f}) \ , \tag{56}$$

and $\widehat{M}_f$ is a real matrix with a form

$$\widehat{M}_f = \begin{pmatrix} 0 & a_f & a_f \\ a_f & b_f & b_f x_f \\ a_f & b_f x_f & b_f \end{pmatrix} , \tag{57}$$

[a_f and b_f have been redefined by including the mixing coefficients, differently from those in the expressions (54)]. Here, note that the parameters δ_i^f are phenomenological ones. If we still require S_2 symmetry, we obtain $\delta_2^f = \delta_3^f$. If we assume an "extended $2 \leftrightarrow 3$ symmetry" with a phase conversion [14], we can obtain $\delta_2^f \neq \delta_3^f$, but the $\mathrm{SU}(2)_L$ symmetry still requires $\delta_3 - \delta_2 \equiv (\delta_3^u - \delta_3^d) - (\delta_2^u - \delta_2^d) = 0$. On the other, in this model, the nonvanishing value of $\delta_3 - \delta_2$ is essential to give a nonvanishing value of $|V_{cb}|$, because it is given by $|V_{cb}| \simeq \sin(\delta_3 - \delta_2)/2$ in the model. Thus, in this model, we cannot obtain reasonable CKM and MNS mixing matrices without breaking the $\mathrm{SU}(2)_L$ symmetry (i.e. $\delta_3 - \delta_2 \neq 0$). For currently proposed phenomenological mass matrix models with a flavor symmetry breaking, it is important to check whether the mass matrix forms with a broken flavor symmetry are still invariant or not under the $\mathrm{SU}(2)_L$ symmetry.

Finally, we would like to comment on the results (42) and (43) in the case of $m_{\nu 1} = m_{\nu 2}$. This suggests a possibility that we can reasonable understand the observed smallness of $|(U_\ell)_{13}|$ [15] and $|(V_q)_{ub}|$ [16] if we consider a model with a flavor symmetry of Z_2 type, $(U_X)^2 = \mathbf{1}$, and with $m_{\nu 1} = m_{\nu 2}$ and $m_{d1} = m_{d2}$ at $\mu > M_X$. This will give a promising clue to possible features of the unbroken flavor symmetry.

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